

Quiz #9: EM Oscillations and Alternating Current

Problem 1 (1 point)

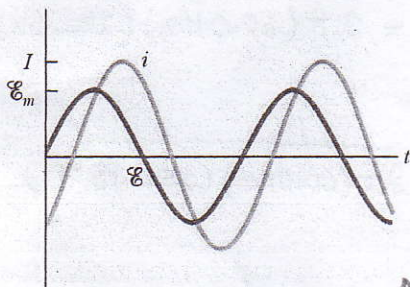
An RLC series circuit is driven by a sinusoidal emf with angular frequency ω_d . If ω_d is increased without changing the amplitude of the emf, the current amplitude increases. This means that:

- D**
- a) $\omega_d L > R$
 - b) $\omega_d L < R$
 - c) $\omega_d L > 1/\omega_d C$
 - d) $\omega_d L < 1/\omega_d C$**
 - e) $\omega_d L = 1/\omega_d C$

↓ the circuit is getting closer to resonance ($X_L = X_C$) when ω_d is increased so $\omega_d L < 1/\omega_d C$

Problem 2 (2 points)

The figure below shows the current i and driving emf ε for a series RLC circuit driven at frequency f_d . What effect (increase, decrease, or no change) would each of the following changes have on (a) the current amplitude I and (b) the phase angle ϕ ?

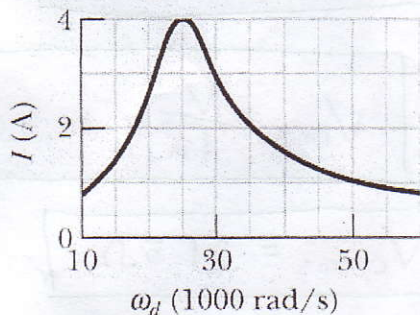


	Effect on I	Effect on ϕ
Increase L	decrease	increase
Increase R	decrease	decrease
Increase C	decrease	increase
Increase f_d	decrease	increase

since i lags ε , circuit is mainly inductive ($\varepsilon \angle i$)
so $X_L < X_C$, $\omega_d > \omega$, $\phi > 0$

Problem 3 (2 points)

The current amplitude I versus driving frequency ω_d for a series RLC circuit is given in the figure below (Note: the driving angular frequency is given in units of 1000 rad/s). The inductance is 175 μH and the emf amplitude is $\varepsilon_m = 125$ V. What are the values of (a) C and (b) R ?



note: from the graph, the resonant angular frequency is $\omega = 25 \times 10^3 \text{ rad/s}$

$$\omega = \frac{1}{\sqrt{LC}} \rightarrow \omega^2 = \frac{1}{LC} \rightarrow C = \frac{1}{\omega^2 L}$$

$$C = \frac{1}{(25 \times 10^3 \text{ rad/s})^2 (175 \times 10^{-6} \text{ H})}$$

$$C = 9.14 \times 10^{-6} \text{ F} = 9.14 \mu\text{F}$$

(b) at resonance ($X_L = X_C$), $Z = R$

$$I = \varepsilon_m / Z = \varepsilon_m / R \rightarrow R = \varepsilon_m / I = \frac{125 \text{ V}}{4.0 \text{ A}} = 31.3 \Omega$$

Problem 4 (2 points)

In an oscillating LC circuit, $L = 30.0 \text{ mH}$ and $C = 7.50 \mu\text{F}$. At time $t = 1.50 \text{ s}$, the current is 9.0 mA and the charge on the capacitor is $3.50 \mu\text{C}$. (a) What is the maximum charge on the capacitor? (b) What is the maximum current?

$$\begin{aligned}
 L &= 30.0 \text{ mH} & U_{\text{total}} &= \frac{1}{2} \frac{q^2}{C} + \frac{1}{2} L i^2 \\
 C &= 7.50 \mu\text{F} & &= \frac{1}{2} \frac{(3.50 \times 10^{-6} \text{ C})^2}{(7.50 \times 10^{-6} \text{ F})} + \frac{1}{2} (30.0 \times 10^{-3} \text{ H}) (9.0 \times 10^{-3} \text{ A})^2 \\
 i &= 9.0 \text{ mA} & &= 2.03 \times 10^{-6} \text{ J} \\
 q &= 3.50 \mu\text{C}
 \end{aligned}$$

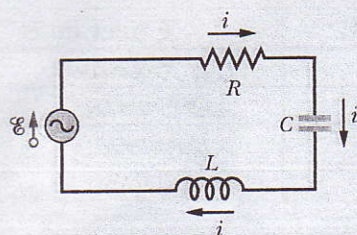
$$\begin{aligned}
 \text{(a)} \quad U_{\text{total}} &= \frac{1}{2} \frac{Q^2}{C} \rightarrow Q = \sqrt{2UC} = \sqrt{2(2.03 \times 10^{-6} \text{ J})(7.50 \times 10^{-6} \text{ F})} \\
 &= \boxed{5.52 \mu\text{C}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad U_{\text{total}} &= \frac{1}{2} L I^2 \rightarrow I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2(2.03 \times 10^{-6} \text{ J})}{30.0 \times 10^{-3} \text{ H}}} \rightarrow \boxed{I = 11.6 \text{ mA}}
 \end{aligned}$$

Problem 5 (3 points)

In the figure below, let $R = 100.0 \Omega$, $C = 25.0 \mu\text{F}$, $L = 200.0 \text{ mH}$, $f_d = 60.0 \text{ Hz}$, and $\mathcal{E}_m = 30.0 \text{ V}$.

(a) What is the maximum current in the circuit? (b) What are the rms voltages across the resistor, capacitor, and the inductor?



$$X_L = \omega_d L = (2\pi f_d) L = 2\pi(60.0 \text{ Hz})(200.0 \times 10^{-3} \text{ H})$$

$$= 75.4 \Omega$$

$$X_C = \frac{1}{\omega_d C} = \frac{1}{(2\pi f_d) C} = \frac{1}{2\pi(60.0 \text{ Hz})(25.0 \times 10^{-6} \text{ F})}$$

$$= 106 \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(100.0 \Omega)^2 + (75.4 \Omega - 106 \Omega)^2} = \underline{104.6 \Omega}$$

$$\begin{aligned}
 \text{a)} \quad I &= \mathcal{E}_m / Z = \frac{30.0 \text{ V}}{104.6 \Omega} \rightarrow \boxed{I = 0.287 \text{ A} = 287 \text{ mA}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad V_R &= IR = (0.287 \text{ A})(100.0 \Omega) = 28.7 \text{ V} & V_{\text{rms}} &= \frac{V}{\sqrt{2}} = 20.3 \text{ V}
 \end{aligned}$$

$$V_C = IX_C = (0.287 \text{ A})(106 \Omega) = 30.4 \text{ V} \quad \boxed{V_{C,\text{rms}} = 21.5 \text{ V}}$$

$$V_L = IX_L = (0.287 \text{ A})(75.4 \Omega) = 21.6 \text{ V} \quad \boxed{V_{L,\text{rms}} = 15.3 \text{ V}}$$